

复杂几何域中不可压流动的多重网格计算

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【摘要】 本文首次将多重网格中的全近似格式 FAS 用到一般曲线坐标系下交错网格布局的 SIM-
PLEC 算法中, 采用三个流动结构: 渐扩平面通道、收扩管流和轴对称弯曲管流作为算例, 通过与单
网格在不同 Re 数、不同网格点数和不同几何结构上的迭代性能的对比, 证明多重网格克服了曲线
坐标系下单网格 SIM PLEC 算法的不足, 大大提高收敛速度, 节省 CPU 时间。

主题词: 不可压缩流动 气动力计算 曲线坐标 收敛

分类号: V231.3

1 前 言

多重网格技术较少地受到网格节点数的影响, 是提高收敛速度的一个有效办法。它可以把
现有计算程序的计算速度提高 1~2 个数量级, 从而特别适合于应用在超大型工程数值计算问
题中。在不可压流求解过程中, 主要的差别在于压力和速度的耦合问题, 分两大类。一类称为
耦合求解法, Vanka 将多重网格应用到直角坐标系下^[1]和曲线坐标系下^[2]速度与压力耦合的
SCGS 法中。然而直接耦合法所需内存大, 额外的偏微分方程不易加入, 而且 SCGS 属于点算
子求解, 对各向异性问题收敛效果不好, 为改进点算子求解, 刘超群^[3]采用近似线盒迭代法与
多重网格结合。另一类称为非耦合求解法, 以 SIMPLE 算法为代表, 意即求解压力耦合方程的
半隐式方法, 在不可压流动计算中得以广泛应用。Smith^[4, 5]将多重网格应用到同位网格布局的
压力修正法的层流流动中。然而同位网格必须给定压力边界值, 且对松弛因子的敏感度较高,
而交错网格却能很好避免上述问题, 能更自然地保证流动的守恒性, 但编程复杂, 特别在多重
网格中限制与插值运算更不易完成。这些不利使多重网格在交错网格布局的 SIMPLE 方法的
应用甚少。本文是在上述方面的一个有益尝试, 即在贴体坐标下采用多重网格法计算复杂几何
域中不可压内流的层流和湍流流动, 以笛卡尔速度为自变量, 交错网格布局, 与分离式求解的
SIMPLEC 算法有力结合, 得到了满意的结果。

2 控制方程及数值求解

对于曲线坐标下的连续、动量及湍流方程, 基本微分方程以强守恒方式表示为统一形式:

$$\frac{1}{J} \frac{\partial}{\partial \xi} (\rho U \Phi) + \frac{1}{J} \frac{\partial}{\partial \eta} (\rho V \Phi) = \frac{1}{J} \frac{\partial}{\partial \xi} \left[\frac{\Gamma_{\Phi}}{J} (\alpha \Phi_{\xi} - \beta \Phi_{\eta}) \right] + \frac{1}{J} \frac{\partial}{\partial \eta} \left[\frac{\Gamma_{\Phi}}{J} (-\beta \Phi_{\xi} + \alpha \Phi_{\eta}) \right] + S^{\Phi}(\xi, \eta) \quad (1)$$

其中 $U = uy_{\eta} - vx_{\eta}$, $V = vx_{\xi} - uy_{\xi}$, $\alpha = x_{\eta}^2 + y_{\eta}^2$

$$\gamma = x_{\xi}^2 + y_{\xi}^2, \quad \beta = x_{\xi}x_{\eta} + y_{\xi}y_{\eta}, \quad J = x_{\xi}y_{\eta} - x_{\eta}y_{\xi}$$

在方程(1)中 Φ 取1时,表示连续方程;取 u, v 和 w 表示动量方程;取 k 和 ϵ 分别表示湍动能方程和湍流耗散率方程。差分,各变量布局为交错网格,速度 u 和 v 位于控制体表面, p 及其它标量位于控制体中心。控制方程采用有限容积法离散,混合差分格式用来计算对流及扩散项,离散方程基本形式为:

$$a_p \Phi_p = \sum_{nb} a_{nb} \Phi_{nb} + S^{\Phi} \quad (2)$$

其中: Φ_{nb} 为 Φ 的邻近值, S^{Φ} 为源项。

在各层网格上的各变量采用SIMPLEC算法的分离式方法求解,即每个方程按顺序求解。在某一网格层上:(1)假定一压力场,依次求解动量方程,得到各速度。(2)求解压力修正方程,得到 p' 。(3)基于 p' 修正各速度和压力。(4)如果在最细网格,求解标量(如 w, k 和 ϵ)。

单网格仅是简单地反复运行上述4步,直到残差达到规定值,而多重网格则是对上述几步在各层网格上执行,直到最细网格的残差达到规定,特别注意的是,对于湍流中 k, ϵ 及二维旋流的 w 速度,它们的求解仅在最细网格上求解,不参与粗网格的迭代。

3 多重网格方法

多重网格法采用细网格可去掉高频误差分量,粗网格可有效去除那些光滑的低频分量,当采用几层网格时,各种频率的误差可以得到比较均匀的衰减,因而加快了迭代收敛的速度。全近似格式FAS适用于求解非线性方程如 $N-S$ 方程,其方法简述如下:

在细网格上控制方程可写成:

$$L^k \Phi^k = F^k \quad (3)$$

其中: L 为非线性算子,由对流和扩散项组成, Φ 为求解变量, F 代表源项。在细网格上用TDMA迭代数次,得到近似解 Φ^k ,而后转换到下一层粗网格,其方程为:

$$L^{k-1} \Phi^{k-1} = L^{k-1} I_k^{k-1} \Phi^k + \hat{I}_k^{k-1} (F^k - L^k \Phi^k) \quad (4)$$

其中: I_k^{k-1} 为对变量本身的限制算子,是把细网格上的变量限制到粗网格上; $\hat{I}_k^{k-1}$ 为对残差的限制算子。在粗网格上迭代数次,然后再到下一层粗网格,直到最粗网格。接着修正粗网格:

$$\Phi_{ew}^k = \Phi_{ld}^k + I_{k-1}^k (\Phi^{k-1} - I_k^{k-1} \Phi_{ld}^k) \quad (5)$$

其中: I_{k-1}^k 为插值算子,是把粗网格上的变量插值到细网格上,以新的 Φ^k 为初值,在较细网格上再迭代,直至最细网格,即完成一步FAS迭代。

变量与残差的插值与限制运算在多重网格计算中是相当重要的。由于采用交错网格, u, v 和 p 的插值算子和限制算子各不相同,非正交曲线坐标系的采用使得普通的双线性插值和六点平均限制不再适用,必须考虑采用面积加权法来进行插值和限制运算。

4 结果和讨论

为验证多重网格在内流流动中的收敛特性,本文计算了三个算例,即渐扩平面通道流动,收扩管道平面流动和弯曲管道轴对称流动。

4.1 渐扩平面通道流动

渐扩平面通道流动计算采用贴体坐标系, 65×33 网格, $Re = 100$ 的层流流动,其外形及网格图如图1。图2所示为单网格与多重网格在不同网格数时的收敛过程对比,多重网格达到收

表 1 单网格和多重网格迭代数与收敛时间对比

网格大小	65×33		129×65	
网格层数	1	5	1	5
迭代数	340	49	1156	120
CPU 时间(s)	332	191	10519	4822

一些时间,然而由于迭代数目的大大减少,总 CPU 时间上还是显示了一定优势。

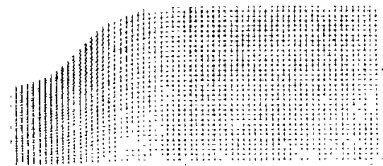


图 1 渐扩平面通道网格图

4.2 收扩管道平面流动

带有两个圆弧凸块的收扩管道平面流动采用 65×33 网格,其外形及网格图如图 3,计算了 $Re=100$ 的层流流动状态及 $Re=10^5$ 的湍流流动状态。图 4 为 $Re=100$ 时流动速度等值线,

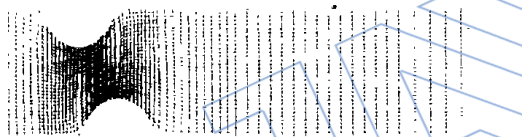


图 3 收扩管流网格图

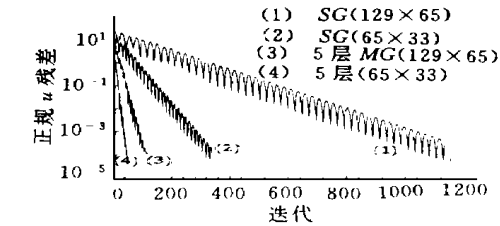


图 2 不同网格数收敛过程对比



图 4 收扩管流速度等值线($Re=100$)

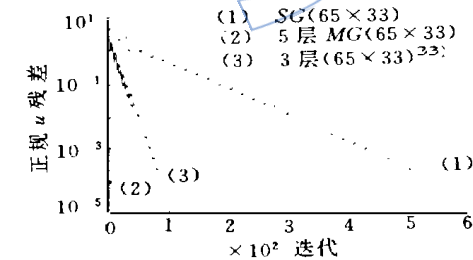


图 5 不同层数收敛过程对比

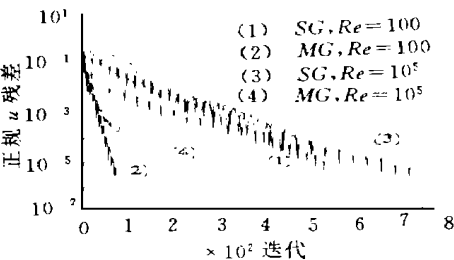


图 6 不同 Re 数收敛过程对比

在两凸块后出现回流区。图 5 为 $Re=100$ 时不同层数收敛过程对比,粗细网格层数对多重网格收敛影响不大,当达到一定层数后,光滑分量即能消除。图 6 为不同 Re 数,即层流与湍流收敛过程对比,在单网格与多重网格上,湍流收敛速率都较层流要慢,但单网格由于非正交项的出现,残差衰减振荡剧烈,而多重网格由于在粗网格上低频分量的有效去除,振荡大大减弱。

4.3 弯曲管道轴对称流动

弯曲管道轴对称流动计算采用贴体坐标,33×17 网格,其外形和网格图如图 7,计算了 Re

$= 3.58 \times 10^6$ 有旋湍流流动状态(入口旋角为 36°)。图 8 对比了单网格与多网格质量残差及湍

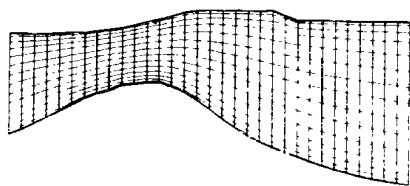


图 7 轴对称弯曲管流网格图

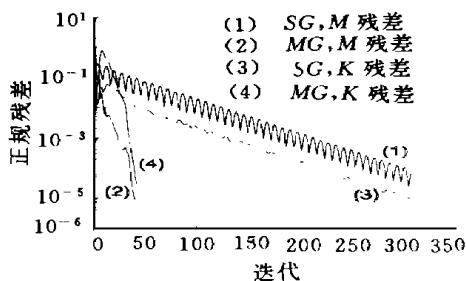


图 8 收敛过程对比



图 9 单网格速度等值线



图 10 多重网格速度等值线

动能残差的下降趋势, 又由于湍流方程中 k 和 ϵ 并未采用多重网格迭代, 仅在最细网格上计算, 所以多重网格中湍动能残差在开始有振荡, 随着 u, v 和 p 残差的快速下降, k 残差也快速下降。图 9 和图 10 所示为单网格与多重网格计算的速度等值线, 对比可看出两者一致, 说明在达到相同收敛准则后, 单网格与多重网格计算精度是一致的。

5 结 论

本文完成了多重网格法在 SIMPLEC 算法中的应用, 解决了交错网格布局的复杂性, 通过曲线坐标系下不可压层流和湍流的各算例计算, 显示出多重网格在收敛速度及 CPU 时间方面的良好性能。单网格随网格节点数的增加迭代次数大大增加, 而多重网格却相对受网格节点数的影响小。这表明多重网格不仅可以快速求解流动问题, 而且还可以在提高精度的同时不用牺牲收敛速度, 尤其在非正交曲线坐标系中能较好抑制残差的振荡现象。

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THREE- DIMENSIONAL BLADING AND ITS INFLUENCE ON BLADE AERODYNAMIC LOADING

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ABSTRACT

The effects of turbomachinery blade bow and sweep on aerodynamic loading distribution of the blade are investigated. A general definition of sweep angle is given in the case of turbomachinery. By means of linearized small perturbation theory, the effects of each factor are analyzed respectively, then a numerical simulation for three- dimensional viscous flow is used to verify and elaborate the analytical results. It is shown that the blade bow, twist and leading edge sweep are effective in adjusting the loading distribution on the blade surface. It is possible to make rational and effective use of the three dimensional blading to adjust the loading distributions both spanwise and chordwise, it thus provides a set of means for blading optimization.

Key Words Turbomachine Blades Three dimensional flow

MEASURING TECHNIQUE OF THREE- DIMENSIONAL PERIODIC UNSTEADY FLOWFIELD IN TURBOMACHINERY

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ABSTRACT

Measuring technique of three- dimensional periodic unsteady flow- field in turbomachinery, including velocity and pressure fields has been developed. A more effective empirical correlation is proposed to express the sensitivity of single slant hot- wire anemometer output to flow velocity, sensor yaw angle and pitch angle. An improved single slant hot- wire three- dimensional unsteady velocity measuring technique is provided, by which the three- dimensional velocity can be measured with very high accuracy and efficiency within a wide range of the flow angle and velocity. Based on measured three- dimensional velocity field and two- dimensional pressure field, measured by pressure transducer fixed on the casing of turbomachinery, a simple experimental- numerical method has been developed to measure the three- dimensional pressure field. With the help of two- dimensional traverse mechanism, high speed data acquisition system and phase- locked sampling and ensemble averaging technique, the three- dimensional periodic unsteady flowfields at the exit of an axial fan have been successfully measured, and its structure has been revealed.

Key Words Turbomachines Flowfields Measurements

A MULTIGRID COMPUTATION OF INCOMPRESSIBLE FLOW IN COMPLEX GEOMETRIES

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ABSTRACT

A procedure using a Full Approximate Scheme (FAS) multigrid is

developed in SIM PLEC algorithm with a layout of staggered grid in generalized curvilinear coordinates. This method is applied to three flow problems as test tasks. The comparison between the iteration performance of the Multi- Grid (MG) and Single- Grid (SG) methods in different grid sizes, Reynolds number and geometries demonstrates that the multigrid method improves convergence rate, reduces CPU time, and overcomes the deficiency of single - grid SIM PLEC algorithm in generalized curvilinear coordinates.

Key Words Incompressible flow Aerodynamic computation Curvilinear coordinates
Convergence

EFFECTS OF GEOMETRIC AND AERODYNAMIC PARAMETERS ON INTERACTION OF WAVES AND VORTEX ON PERFORATED PLATE

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ABSTRACT An experimental investigation has been carried out on a kind of perforated liner with a bias flow through the apertures, according to Howe's vortex sound theory. The attention was focused on the effects of the aperture size, the cavity depth and the perforated plate thickness on the impedance of the liner. The algorithm of the reactance is modified for the liner with thick perforated plate, based on the analysis of experimental results. The perforated liners with sucking and blowing bias flow are compared and the same conclusions are drawn out.

Key Words Geometry Aerodynamic forces Sound reduction

AN EXPERIMENTAL INVESTIGATION ON ARC SKEWED SLOT CASING TREATMENT

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ABSTRACT Different geometrical designs of arc skewed slot casing treatments have been investigated experimentally to research their influences on stall margin and operating efficiency. A study on the mechanism of the casing treatment was carried out on the basis of three dimension flow field measurement. The comparison of different casing treatments revealed that the best configuration of an arc skewed slot casing treatment improved the stall margin by 27%, and the peak efficiency by 1%. Compared with an axial skewed slot casing treatment, the arc skewed one not only made an improvement in stall margin, but also ended the history that the casing treatment had to pay a loss of efficiency in order to improve stall margin.

Key Words Axial fan Treatment Casing Flow measurement